<9.11~9.15절>

9.117 $m = \rho\, t\, A = \rho\, t\left(\frac{1}{2}bh\right) \Rightarrow \rho\, t = \frac{2m}{bh} \Rightarrow I^{mass} = \rho\, t\, I^{area} = \frac{2m}{bh}\; I^{area}$

(a) $\bar{I}^{area}_{AA'} = 2\left[\frac{1}{12}(h)\left(\frac{b}{2}\right)^3\right] = \frac{1}{48}b^3h$

$\bar{I}^{mass}_{AA'} = \frac{2m}{bh}\;\bar{I}^{area}_{AA'} = \frac{2m}{bh}\;\frac{1}{48}b^3h = \frac{1}{24}mb^2$

$\bar{I}^{area}_{BB'} = \frac{1}{36}bh^3$

$\bar{I}^{mass}_{BB'} = \frac{2m}{bh}\;\bar{I}^{area}_{BB'} = \frac{2m}{bh}\;\frac{1}{36}bh^3 = \frac{1}{18}mh^2$

(b) $I^{mass}_{CC'} = \bar{I}^{mass}_{AA'} + \bar{I}^{mass}_{BB'} = \frac{1}{24}mb^2 + \frac{1}{18}mh^2 = \frac{m}{72}\left(3b^2+4h^2\right)$

9.118 (9.117(a)로부터 $\bar{I}^{mass}_{AA'} = \frac{1}{24}mb^2,\quad \bar{I}^{mass}_{BB'} = \frac{1}{18}mh^2$)

$\bar{I}^{mass}_{DD'} = \bar{I}^{mass}_{AA'} + m\,d^2 = \frac{1}{24}mb^2 + m\,d^2 = \frac{1}{24}m\left(b^2+24d^2\right)$

$\bar{I}^{mass}_{EE'} = \bar{I}^{mass}_{BB'} + m\,d^2 = \frac{1}{18}mh^2 + m\,d^2 = \frac{1}{18}m\left(h^2+18d^2\right)$

9.119 $m = \rho\, t\, A = \rho\, t\left[(2a)(a)+\frac{1}{2}(2a)(a)\right] = 3\,\rho\, t\, a^2 \Rightarrow \rho\, t = \frac{m}{3a^2}$

$I^{mass} = \rho\, t\, I^{area} = \frac{m}{3a^2}\; I^{area}$

(a) 사각형 $I^{area}_{x1} = \frac{1}{3}(2a)(a)^3 = \frac{2}{3}a^4,$ 삼각형 $I^{area}_{x2} = \frac{1}{12}(2a)(a)^3 = \frac{1}{6}a^4$

$I^{area}_x = I^{area}_{x1} + I^{area}_{x2} = \frac{2}{3}a^4 + \frac{1}{6}a^4 = \frac{5}{6}a^4$

$I^{mass}_x = \frac{m}{3a^2}\; I^{area} = \frac{m}{3a^2}\;\frac{5}{6}a^4 = \frac{5}{18}ma^2$

(b) 사각형 $I^{area}_{z1} = \frac{1}{3}(a)(2a)^3 = \frac{8}{3}a^4$

삼각형 $I^{area}_{z2} = \frac{1}{36}(a)(2a)^3 + \left[\frac{1}{2}(a)(2a)\right]\left(2a+\frac{2}{3}a\right)^2 = \left(\frac{2}{9}+\frac{64}{9}\right)a^4 = \frac{22}{3}a^4$

$I^{area}_z = I^{area}_{z1} + I^{area}_{z2} = \frac{8}{3}a^4 + \frac{22}{3}a^4 = 10\,a^4$

$I^{mass}_z = \frac{m}{3a^2}\; I^{area} = \frac{m}{3a^2}\; 10\,a^4 = \frac{10}{3}ma^2$

$I^{mass}_y = I^{mass}_x + I^{mass}_z = \frac{5}{18}ma^2 + \frac{10}{3}ma^2 = \left(\frac{5}{18}+\frac{10}{3}\right)ma^2 = \frac{65}{18}ma^2$

$= 3.61\; m\,a^2$

9.132 밀도 ρ

원기둥(단면 반지름 r, 높이 h)의 질량 관성모멘트 $= \frac{1}{2}(\rho\pi r^2 h)r^2 = \frac{1}{2}\rho\pi r^4 h$

$\bar{I}_{AA'} = \frac{1}{2}\rho\pi a_2^4 h - \frac{1}{2}\rho\pi a_1^4 h = \frac{1}{2}\rho\pi\left(a_2^4 - a_1^4\right)h$

질량 $m = \rho\pi\left(a_2^2 - a_1^2\right)h$

(a) $I_{BB'} = \bar{I}_{AA'} + m\,a_1^2 = \frac{1}{2}\rho\pi\left(a_2^4 - a_1^4\right)h + \rho\pi\left(a_2^2 - a_1^2\right)h\ a_1^2$

$= \frac{1}{2}\rho\pi h\left(a_2^4 + 2a_2^2 a_1^2 - 3a_1^4\right)$

(b) $\frac{d}{da_1}I_{BB'} = \frac{d}{da_1}\left[\frac{1}{2}\rho\pi h\left(a_2^4 + 2a_2^2 a_1^2 - 3a_1^4\right)\right]$

$= \frac{1}{2}\rho\pi h\left(4a_2^2 a_1 - 12a_1^3\right) = 2\rho\pi h a_1\left(a_2^2 - 3a_1^2\right) = 0$

$\Rightarrow \quad a_1\left(a_2^2 - 3a_1^2\right) = 0, \quad a_1 \neq 0 \quad \Rightarrow \quad a_2^2 - 3a_1^2 = 0 \quad \Rightarrow \quad a_1 = \frac{1}{\sqrt{3}}a_2$

(c) $I_{BB'}\Big|_{a_1 = \frac{a_2}{\sqrt{3}}} = \frac{1}{2}\rho\pi h\left[a_2^4 + 2a_2^2\left(\frac{a_2}{\sqrt{3}}\right)^2 - 3\left(\frac{a_2}{\sqrt{3}}\right)^4\right]$

$= \frac{1}{2}\rho\pi h a_2^4\left[1 + \frac{2}{3} - \frac{1}{3}\right] = \frac{2}{3}\rho\pi h a_2^4$