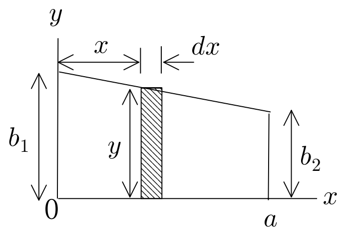
정 역 학　　연습문제 풀이　　제9장　　(2009년)

<9.1~9.5절>

9.5



$$y = b_1 + \frac{b_2 - b_1}{a}x$$

$$dI_x = \frac{1}{3}y^3\,dx = \frac{1}{3}\left(b_1 + \frac{b_2 - b_1}{a}x\right)^3 dx$$

$$I_x = \int_0^a \frac{1}{3}\left(b_1 + \frac{b_2 - b_1}{a}x\right)^3 dx$$

$$= \frac{1}{3}\left[\frac{1}{4}\frac{a}{b_2 - b_1}\left(b_1 + \frac{b_2 - b_1}{a}x\right)^4\right]_0^a$$

$$= \frac{1}{12}\frac{a}{b_2 - b_1}\left(b_2^4 - b_1^4\right) = \frac{1}{12}a\,(b_2 + b_1)\left(b_2^2 + b_1^2\right)$$

9.16

$$y_1 = k\sqrt{x}$$

$$(a,\ 2b) \quad \Rightarrow \quad 2b = k\sqrt{a} \quad \Rightarrow \quad k = \frac{2b}{\sqrt{a}}$$

$$\Rightarrow \quad y_1 = \frac{2b}{\sqrt{a}}\sqrt{x}$$

$$y_2 = c\,x$$

$$(a,\ b) \quad \Rightarrow \quad b = c\,a \quad \Rightarrow \quad c = \frac{b}{a}$$

$$\Rightarrow \quad y_2 = \frac{b}{a}x$$

$$A = 2\int_0^a (y_1 - y_2)\,dx = 2\int_0^a \left(\frac{2b}{\sqrt{a}}x^{\frac{1}{2}} - \frac{b}{a}x\right)dx = 2b\left[\frac{4}{3\sqrt{a}}x^{\frac{3}{2}} - \frac{1}{2a}x^2\right]_0^a$$

$$= 2b\left(\frac{4}{3\sqrt{a}}a^{\frac{3}{2}} - \frac{1}{2a}a^2\right) = \frac{5}{3}ab$$

$$dI_x = \frac{1}{3}y_1^3\,dx - \frac{1}{3}y_2^3\,dx = \frac{1}{3}\left(y_1^3 - y_2^3\right)dx = \frac{1}{3}\left(\frac{8b^3}{a^{3/2}}x^{\frac{3}{2}} - \frac{b^3}{a^3}x^3\right)dx$$

$$I_x = 2\int_0^a \frac{1}{3}\left(\frac{8b^3}{a^{3/2}}x^{\frac{3}{2}} - \frac{b^3}{a^3}x^3\right)dx = \frac{2b^3}{3}\int_0^a\left(\frac{8}{a^{3/2}}x^{\frac{3}{2}} - \frac{1}{a^3}x^3\right)dx$$

$$= \frac{2b^3}{3}\left[\frac{16}{5a^{3/2}}x^{\frac{5}{2}} - \frac{1}{4a^3}x^4\right]_0^a = \frac{2b^3}{3}\left(\frac{16}{5a^{3/2}}a^{\frac{5}{2}} - \frac{1}{4a^3}a^4\right) = \frac{2}{3}\left(\frac{16}{5} - \frac{1}{4}\right)ab^3$$

$$= \frac{59}{30}ab^3$$

$$k_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{\frac{59}{30}ab^3}{\frac{5}{3}ab}} = \sqrt{\frac{59}{50}}\,b = 1.086\ b$$

9.22 $A = (2a)(3b) - \frac{1}{2}(2a)(b) = 5ab$

<방법1 : 사각형, 삼각형 공식 사용>

$I_{x1} = \frac{1}{3}(2a)(2b)^3 = \frac{16}{3}ab^3$

$I_{x2} = I_{x3} = \frac{1}{12}ab^3$

$I_x = I_{x1} + I_{x2} + I_{x3} = \frac{16}{3}ab^3 + 2\left(\frac{1}{12}ab^3\right) = \frac{11}{2}ab^3$

$I_{y1} = \frac{1}{12}(2a)^3(3b) = 2a^3b$

$I_{y2} = I_{y3} = \frac{1}{12}a^3b$

$I_y = I_{y1} - I_{y2} - I_{y3} = 2a^3b - 2\left(\frac{1}{12}a^3b\right) = \frac{11}{6}a^3b$

polar moment of inertia

$J_P = I_x + I_y = \frac{11}{2}ab^3 + \frac{11}{6}a^3b = \frac{11}{6}ab\left(a^2+3b^2\right)$

polar radius of gyration

$k_P = \sqrt{\frac{J_P}{A}} = \sqrt{\frac{\frac{11}{6}ab\left(a^2+3b^2\right)}{5ab}} = \sqrt{\frac{11}{30}\left(a^2+3b^2\right)}$

<방법2 : 사각형 공식, 삼각형 정적분>

$I_{x1} = \frac{1}{3}(2a)(2b)^3 = \frac{16}{3}ab^3$

$y = -\frac{b}{a}x$

$dI_{x2} = \frac{1}{3}(-y)^3dx = \frac{1}{3}\left(\frac{b}{a}x\right)^3dx$

$I_{x3} = I_{x2} = \int dI_{x2} = \int_0^a \frac{b^3}{3a^3}x^3\,dx = \frac{b^3}{3a^3}\left[\frac{1}{4}x^4\right]_0^a = \frac{1}{12}ab^3$

$I_x = I_{x1} + I_{x2} + I_{x3} = \frac{16}{3}ab^3 + 2\left(\frac{1}{12}ab^3\right) = \frac{11}{2}ab^3$

$I_{y1} = \frac{1}{12}(2a)^3(3b) = 2a^3b$

$x = -\frac{a}{b}y$

$dI_{y2} = \frac{1}{3}x^3(-dy) = \frac{1}{3}\left(\frac{a}{b}y\right)^3dy$

$I_{y3} = I_{y2} = \int dI_{y2} = \int_0^{-b} \frac{a^3}{3b^3}y^3\,dy = \frac{a^3}{3b^3}\left[\frac{1}{4}y^4\right]_0^{-b} = \frac{1}{12}a^3b$

$I_y = I_{y1} - I_{y2} - I_{y3} = 2a^3b - 2\left(\frac{1}{12}a^3b\right) = \frac{11}{6}a^3b$

polar moment of inertia

$$J_P = I_x + I_y = \frac{11}{2}ab^3 + \frac{11}{6}a^3b = \frac{11}{6}ab\left(a^2+3b^2\right)$$

polar radius of gyration

$$k_P = \sqrt{\frac{J_P}{A}} = \sqrt{\frac{\frac{11}{6}ab\left(a^2+3b^2\right)}{5ab}} = \sqrt{\frac{11}{30}\left(a^2+3b^2\right)}$$

<방법3 : 2중 적분>

$$I_x = 2\int y^2\,dA = 2\int_0^a\int_{-\frac{b}{a}x}^{2b} y^2\,dy\,dx = 2\int_0^a\left[\frac{1}{3}y^3\right]_{-\frac{b}{a}x}^{2b}dx$$

$$= 2\int_0^a\frac{1}{3}\left\{(2b)^3-\left(-\frac{b}{a}x\right)^3\right\}dx = \frac{2}{3}\left[(2b)^3x+\frac{b^3}{4a^3}x^4\right]_0^a = \frac{2}{3}\left[(2b)^3a+\frac{b^3}{4a^3}a^4\right]$$

$$= \frac{2}{3}\left(8+\frac{1}{4}\right)ab^3 = \frac{11}{2}ab^3$$

$$I_y = 2\int x^2\,dA = 2\int_0^a\int_{-\frac{b}{a}x}^{2b} x^2\,dy\,dx = 2\int_0^a x^2\left[y\right]_{-\frac{b}{a}x}^{2b}dx$$

$$= 2\int_0^a x^2\left\{(2b)-\left(-\frac{b}{a}x\right)\right\}dx = 2\left[(2b)\frac{1}{3}x^3+\frac{b}{4a}x^4\right]_0^a = 2\left[(2b)\frac{a^3}{3}+\frac{b}{4a}a^4\right]$$

$$= 2\left(\frac{2}{3}+\frac{1}{4}\right)a^3b = \frac{11}{6}a^3b$$

polar moment of inertia

$$J_P = I_x + I_y = \frac{11}{2}ab^3 + \frac{11}{6}a^3b = \frac{11}{6}ab\left(a^2+3b^2\right)$$

polar radius of gyration

$$k_P = \sqrt{\frac{J_P}{A}} = \sqrt{\frac{\frac{11}{6}ab\left(a^2+3b^2\right)}{5ab}} = \sqrt{\frac{11}{30}\left(a^2+3b^2\right)}$$