

<5.10~5.11 >

5.92

$$V = \frac{1}{2} \pi a^2 h$$

$$\bar{z} = -\frac{4}{3\pi} a$$

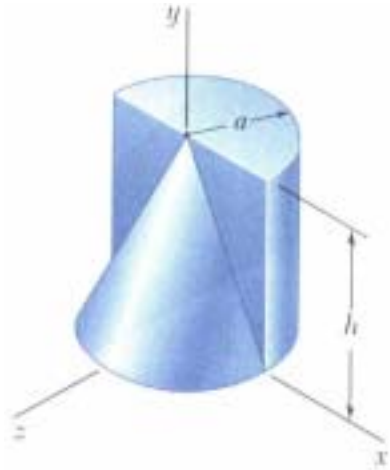
$$V = \frac{1}{2} \left(\frac{1}{3} \pi a^2 h \right) = \frac{1}{6} \pi a^2 h$$

$$\bar{z} = \frac{1}{\pi} a$$

$$V = \frac{1}{2} \pi a^2 h + \frac{1}{6} \pi a^2 h = \frac{2}{3} \pi a^2 h$$

$$(\bar{z} V) = \left(-\frac{4}{3\pi} a \right) \left(\frac{1}{2} \pi a^2 h \right) + \left(\frac{1}{\pi} a \right) \left(\frac{1}{6} \pi a^2 h \right) = -\frac{1}{2} a^3 h$$

$$\bar{Z} = \frac{\Sigma(\bar{z} V)}{\Sigma V} = \frac{-\frac{1}{2} a^3 h}{\frac{2}{3} \pi a^2 h} = -\frac{3}{4\pi} a$$



5.102

$$\bar{X} = 150 \text{ mm}$$

$$A = \frac{1}{2} \pi (150 \text{ mm})^2 = 35,343 \text{ mm}^2$$

$$\bar{y} = (180 \text{ mm}) + (96 \text{ mm}) + \frac{4}{3\pi} (150 \text{ mm}) = 339.7 \text{ mm}$$

$$\bar{z} = 0$$

$$A = \frac{1}{2} \pi (96 \text{ mm})(300 \text{ mm}) = 45,239 \text{ mm}^2$$

$$\bar{y} = (180 \text{ mm}) + \frac{2}{\pi} (96 \text{ mm}) = 241.1 \text{ mm}$$

$$\bar{z} = \frac{2}{\pi} (96 \text{ mm}) = 61.1 \text{ mm}$$

$$A = \sqrt{(180 \text{ mm})^2 + (96 \text{ mm})^2} (300 \text{ mm}) = 61,200 \text{ mm}^2$$

$$\bar{y} = \frac{1}{2} (180 \text{ mm}) = 90 \text{ mm}$$

$$\bar{z} = \frac{1}{2} (96 \text{ mm}) = 48 \text{ mm}$$

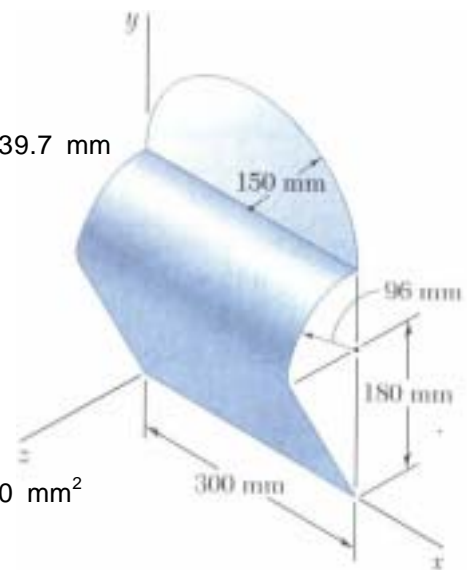
$$A = 35,343 + 45,239 + 61,200 \text{ (mm}^2\text{)} = 141,782 \text{ mm}^2$$

$$(\bar{y} A) = (339.7)(35,343) + (241.1)(45,239) + (90)(61,200) = 28,421,140 \text{ mm}^3$$

$$(\bar{z} A) = (0)(35,343) + (61.1)(45,239) + (48)(61,200) = 5,701,703 \text{ mm}^3$$

$$\bar{Y} = \frac{\Sigma(\bar{y} A)}{\Sigma A} = \frac{28,421,140 \text{ mm}^3}{141,782 \text{ mm}^2} = 200.4 \text{ mm}$$

$$\bar{Z} = \frac{\Sigma(\bar{z} A)}{\Sigma A} = \frac{5,701,703 \text{ mm}^3}{141,782 \text{ mm}^2} = 40.21 \text{ mm}$$



= (150 mm, 200 mm, 40.2 mm)

5.109

$AB ; L = R = 300 \text{ mm}$

$\bar{x} = 0$

$\bar{y} = \frac{1}{2}(300 \text{ mm}) = 150 \text{ mm}$

$\bar{z} = 0$

$DA ; L = 180 \text{ mm} + 100 \text{ mm} = 280 \text{ mm}$

$\bar{x} = \frac{1}{2}(280 \text{ mm}) = 140 \text{ mm}$

$\bar{y} = 0$

$\bar{z} = 0$

$CD ; h = \sqrt{300^2 - 180^2} \text{ mm} = 240 \text{ mm}$

$L = \sqrt{240^2 + 100^2} \text{ mm} = 260 \text{ mm}$

$\bar{x} = (180 \text{ mm}) + \frac{1}{2}(100 \text{ mm}) = 230 \text{ mm}$

$\bar{y} = 0$

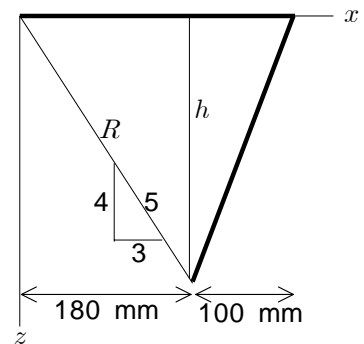
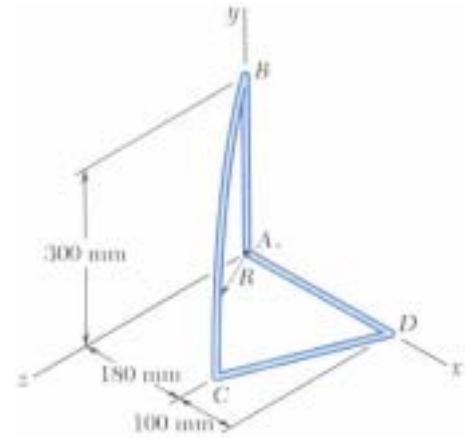
$\bar{z} = \frac{1}{2}(240 \text{ mm}) = 120 \text{ mm}$

$BC ; L = \frac{\pi}{2}(300 \text{ mm}) = 150\pi \text{ mm}$

$\bar{x} = \frac{3}{5} \left[\frac{2}{\pi}(300 \text{ mm}) \right] = \frac{360}{\pi} \text{ mm}$

$\bar{y} = \frac{2}{\pi}(300 \text{ mm}) = \frac{600}{\pi} \text{ mm}$

$\bar{z} = \frac{4}{5} \left[\frac{2}{\pi}(300 \text{ mm}) \right] = \frac{480}{\pi} \text{ mm}$



$L = 300 + 280 + 260 + 150\pi \text{ (mm)} = 1,311.2 \text{ mm}$

$(\bar{x}L) = (0)(300) + (140)(280) + (230)(260) + \left(\frac{360}{\pi}\right)(150\pi) = 153,000 \text{ mm}^2$

$(\bar{y}L) = (150)(300) + (0)(280) + (0)(260) + \left(\frac{600}{\pi}\right)(150\pi) = 135,000 \text{ mm}^2$

$(\bar{z}L) = (0)(300) + (0)(280) + (120)(260) + \left(\frac{480}{\pi}\right)(150\pi) = 103,200 \text{ mm}^2$

$\bar{X} = \frac{\Sigma(\bar{x}L)}{\Sigma L} = \frac{153,000 \text{ mm}^2}{1,311.2 \text{ mm}} = 116.7 \text{ mm}$

$\bar{Y} = \frac{\Sigma(\bar{y}L)}{\Sigma L} = \frac{135,000 \text{ mm}^2}{1,311.2 \text{ mm}} = 103.0 \text{ mm}$

$\bar{Z} = \frac{\Sigma(\bar{z}L)}{\Sigma L} = \frac{103,200 \text{ mm}^2}{1,311.2 \text{ mm}} = 78.7 \text{ mm}$

$= (116.7 \text{ mm}, 103.0 \text{ mm}, 78.7 \text{ mm})$