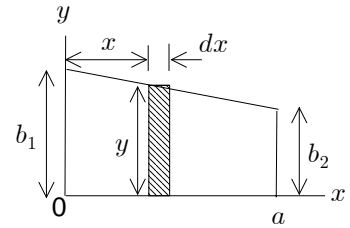


&lt;9.1~9.5 &gt;

$$9.1 \quad y = b_1 + \frac{b_2 - b_1}{a} x$$

$$\begin{aligned} dI_y &= x^2 dA = x^2 y dx = x^2 \left( b_1 + \frac{b_2 - b_1}{a} x \right) dx \\ &= \left( b_1 x^2 + \frac{b_2 - b_1}{a} x^3 \right) dx \end{aligned}$$

$$\begin{aligned} I_y &= \int_0^a \left( b_1 x^2 + \frac{b_2 - b_1}{a} x^3 \right) dx = \left[ \frac{b_1}{3} x^3 + \frac{b_2 - b_1}{4a} x^4 \right]_0^a \\ &= \frac{b_1}{3} a^3 + \frac{b_2 - b_1}{4a} a^4 = \frac{1}{12} a^3 (b_1 + 3b_2) \end{aligned}$$



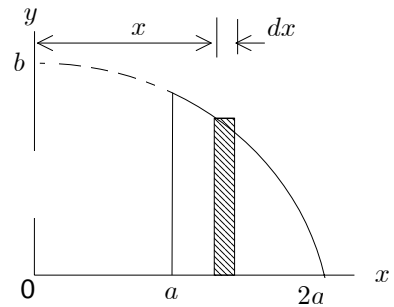
$$9.8 \quad y = kx^2 + c$$

$$(0, b) \quad c = b \quad y = kx^2 + b$$

$$(2a, 0) \quad 0 = k(2a)^2 + b \quad k = -\frac{b}{4a^2}$$

$$y = -\frac{b}{4a^2} x^2 + b = \frac{b}{4a^2} (4a^2 - x^2)$$

$$\begin{aligned} dI_x &= \frac{1}{3} y^3 dx = \frac{1}{3} \left[ \frac{b}{4a^2} (4a^2 - x^2) \right]^3 dx \\ &= \frac{1}{3} \frac{b^3}{64a^6} (4a^2 - x^2)^3 dx = \frac{b^3}{192a^6} (64a^6 - 48a^4x^2 + 12a^2x^4 - x^6) dx \end{aligned}$$



$$\begin{aligned} I_x &= \int dI_x = \int_a^{2a} \frac{b^3}{192a^6} (64a^6 - 48a^4x^2 + 12a^2x^4 - x^6) dx \\ &= \frac{b^3}{192a^6} \left[ 64a^6x - 16a^4x^3 + \frac{12}{5}a^2x^5 - \frac{1}{7}x^7 \right]_a^{2a} \\ &= \frac{b^3}{192a^6} \left\{ 64a^6(2a - a) - 16a^4[(2a)^3 - a^3] + \frac{12}{5}a^2[(2a)^5 - a^5] - \frac{1}{7}[(2a)^7 - a^7] \right\} \\ &= \frac{b^3}{192a^6} \left\{ 64a^7(2 - 1) - 16a^7(8 - 1) + \frac{12}{5}a^7(32 - 1) - \frac{1}{7}a^7(128 - 1) \right\} \\ &= \frac{a b^3}{192} \left[ 64 - 16(7) + \frac{12}{5}(31) - \frac{1}{7}(127) \right] \\ &= 0.0430 a b^3 \end{aligned}$$

$$9.21 \text{ (a)} \quad dI_x = \frac{1}{3} a^3 dx$$

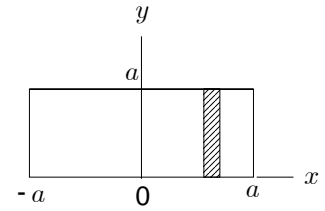
$$I_x = \int dI_x = \int_{-a}^a \frac{1}{3} a^3 dx = \frac{1}{3} a^3 (2a) = \frac{2}{3} a^4$$

$$dI_y = x^2 dA = x^2 a dx$$

$$I_y = \int dI_y = \int_{-a}^a a x^2 dx = a \left[ \frac{1}{3} x^3 \right]_{-a}^a = \frac{2}{3} a^4$$

$$J_O = I_x + I_y = \frac{2}{3} a^4 + \frac{2}{3} a^4 = \frac{4}{3} a^4$$

$$J_O = k_O^2 A \quad k_O = \sqrt{\frac{J_O}{A}} = \sqrt{\frac{\frac{4}{3} a^4}{2a^2}} = \sqrt{\frac{2}{3}} a$$



$$\text{(b)} \quad dI_x = \frac{1}{12} a^3 dx$$

$$I_x = \int dI_x = \int_0^{2a} \frac{1}{12} a^3 dx = \frac{1}{12} a^3 (2a) = \frac{1}{6} a^4$$

$$dI_y = x^2 dA = x^2 a dx$$

$$I_y = \int dI_y = \int_0^{2a} a x^2 dx = a \left[ \frac{1}{3} x^3 \right]_0^{2a} = \frac{8}{3} a^4$$

$$J_O = I_x + I_y = \frac{1}{6} a^4 + \frac{8}{3} a^4 = \frac{17}{6} a^4$$

$$J_O = k_O^2 A \quad k_O = \sqrt{\frac{J_O}{A}} = \sqrt{\frac{\frac{17}{6} a^4}{2a^2}} = \sqrt{\frac{17}{12}} a$$

