

<5.10~5.11 >

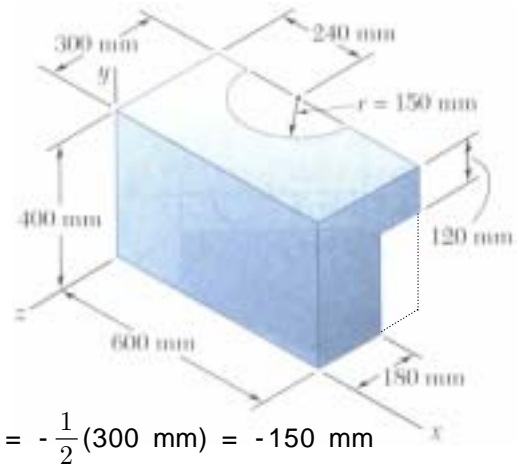
5.91 $V = \frac{1}{2} \pi a^2 h$ $\bar{y} = \frac{1}{2} h$

$V = \frac{1}{2} \left(\frac{1}{3} \pi a^2 h \right) = \frac{1}{6} \pi a^2 h$ $\bar{y} = \frac{1}{4} h$

$V = \frac{1}{2} \pi a^2 h + \frac{1}{6} \pi a^2 h = \frac{2}{3} \pi a^2 h$

$(\bar{y}V) = \left(\frac{1}{2} h \right) \left(\frac{1}{2} \pi a^2 h \right) + \left(\frac{1}{4} h \right) \left(\frac{1}{6} \pi a^2 h \right) = \frac{7}{24} \pi a^2 h^2$

$\bar{Y} = \frac{\Sigma(\bar{y}V)}{\Sigma V} = \frac{\frac{7}{24} \pi a^2 h^2}{\frac{2}{3} \pi a^2 h} = \frac{7}{16} h$



5.100 $A = (600 \text{ mm})(400 \text{ mm}) = 240,000 \text{ mm}^2$

$\bar{x} = \frac{1}{2} (600 \text{ mm}) = 300 \text{ mm},$

$\bar{y} = \frac{1}{2} (400 \text{ mm}) = 200 \text{ mm}, \quad \bar{z} = 0$

$A = (300 \text{ mm})(400 \text{ mm}) = 120,000 \text{ mm}^2$

$\bar{x} = 600 \text{ mm}, \quad \bar{y} = 200 \text{ mm}, \quad \bar{z} = -\frac{1}{2} (300 \text{ mm}) = -150 \text{ mm}$

$A = -(120 \text{ mm})(280 \text{ mm}) = -33,600 \text{ mm}^2$

$\bar{x} = 600 \text{ mm}, \quad \bar{y} = \frac{1}{2} (280 \text{ mm}) = 140 \text{ mm}, \quad \bar{z} = -180 - \frac{1}{2} (120 \text{ mm}) = -240 \text{ mm}$

$A = (600 \text{ mm})(300 \text{ mm}) = 180,000 \text{ mm}^2$

$\bar{x} = 300 \text{ mm}, \quad \bar{y} = 400 \text{ mm}, \quad \bar{z} = -150 \text{ mm}$

$A = -\frac{1}{2} \pi (150 \text{ mm})^2 = -35,343 \text{ mm}^2$

$\bar{x} = 240 \text{ mm}, \quad \bar{y} = 400 \text{ mm},$

$\bar{z} = -300 \text{ mm} + \frac{4}{3\pi} (150 \text{ mm}) = -236.3 \text{ mm}$

$A = 240,000 + 120,000 + -33,600 + 180,000 - 35,343 = 471,057 \text{ mm}^2$

$(\bar{x}A) = (300)(240,000) + (600)(120,000) + (600)(-33,600) + (300)(180,000) + (240)(-35,343) = 169,357,680 \text{ mm}^3$

$(\bar{y}A) = (200)(240,000) + (200)(120,000) + (140)(-33,600) + (400)(180,000) + (400)(-35,343) = 125,158,800 \text{ mm}^3$

$(\bar{z}A) = (0)(240,000) + (-150)(120,000) + (-240)(-33,600) + (-150)(180,000) + (-236.3)(-35,343) = -28,584,449 \text{ mm}^3$

$\bar{X} = \frac{\Sigma(\bar{x}A)}{\Sigma A} = \frac{169,357,680 \text{ mm}^3}{471,057 \text{ mm}^2} = 359.5 \text{ mm}$

$\bar{Y} = \frac{\Sigma(\bar{y}A)}{\Sigma A} = \frac{125,158,800 \text{ mm}^3}{471,057 \text{ mm}^2} = 265.7 \text{ mm}$

$\bar{Z} = \frac{\Sigma(\bar{z}A)}{\Sigma A} = \frac{-28,584,449 \text{ mm}^3}{471,057 \text{ mm}^2} = -60.68 \text{ mm}$

$= (360 \text{ mm}, 266 \text{ mm}, -60.7 \text{ mm})$

$$5.108 \quad AB ; L = [(500 \text{ mm})^2 + (300 \text{ mm})^2 + (750 \text{ mm})^2]^{1/2} = 950 \text{ mm}$$

$$\bar{x} = \frac{1}{2}(500 \text{ mm}) = 250 \text{ mm}, \quad \bar{y} = \frac{1}{2}(750 \text{ mm}) = 375 \text{ mm},$$

$$\bar{z} = \frac{1}{2}(300 \text{ mm}) = 150 \text{ mm}$$

$$BD ; L = 300 \text{ mm}$$

$$\bar{x} = 500 \text{ mm}, \quad \bar{y} = 0, \quad \bar{z} = \frac{1}{2}(300 \text{ mm}) = 150 \text{ mm}$$

$$DO ; L = 500 \text{ mm}$$

$$\bar{x} = \frac{1}{2}(500 \text{ mm}) = 250 \text{ mm}, \quad \bar{y} = 0, \quad \bar{z} = 0$$

$$OA ; L = 750 \text{ mm}$$

$$\bar{x} = 0, \quad \bar{y} = \frac{1}{2}(750 \text{ mm}) = 375 \text{ mm}, \quad \bar{z} = 0$$

$$L = 950 + 300 + 500 + 750 = 2,500 \text{ mm}$$

$$(\bar{x}L) = (250)(950) + (500)(300) + (250)(500) + (0)(750) = 512,500 \text{ mm}^2$$

$$(\bar{y}L) = (375)(950) + (0)(300) + (0)(500) + (375)(750) = 637,500 \text{ mm}^2$$

$$(\bar{z}L) = (150)(950) + (150)(300) + (0)(500) + (0)(750) = 187,500 \text{ mm}^2$$

$$\bar{X} = \frac{\Sigma(\bar{x}L)}{\Sigma L} = \frac{512,500 \text{ mm}^2}{2,500 \text{ mm}} = 205 \text{ mm}$$

$$\bar{Y} = \frac{\Sigma(\bar{y}L)}{\Sigma L} = \frac{637,500 \text{ mm}^2}{2,500 \text{ mm}} = 255 \text{ mm}$$

$$\bar{Z} = \frac{\Sigma(\bar{z}L)}{\Sigma L} = \frac{187,500 \text{ mm}^2}{2,500 \text{ mm}} = 75.0 \text{ mm}$$

$$= (205 \text{ mm}, 255 \text{ mm}, 75.0 \text{ mm})$$