

<9.11~9.15 >

$$9.114 \quad m = \rho t A = \rho t \left(\frac{1}{2} \pi a b \right)$$

$$I^{mass} = \rho t I^{area} = \frac{2m}{\pi a b} I^{area}$$

$$(a) \quad I_{BB'}^{area} = I_x^{area} - A \bar{y}^2 \\ = \frac{1}{8} \pi a b^3 - \left(\frac{1}{2} \pi a b \right) \left(\frac{4b}{3\pi} \right)^2 = \pi a b^3 \left(\frac{1}{8} - \frac{8}{9\pi^2} \right) = \frac{\pi}{8} a b^3 \left(1 - \frac{64}{9\pi^2} \right)$$

$$I_{BB'}^{mass} = \frac{2m}{\pi a b} I_{BB'}^{area} \\ = \frac{2m}{\pi a b} \frac{\pi}{8} a b^3 \left(1 - \frac{64}{9\pi^2} \right) = \frac{1}{4} m b^2 \left(1 - \frac{64}{9\pi^2} \right) = 0.0699 m b^2$$

$$(b) \quad \text{Fig. 9.12} \quad , \quad I_{AA'}^{area} = \frac{1}{8} \pi a^3 b$$

$$I_{AA'}^{mass} = \frac{2m}{\pi a b} I_{AA'}^{area} \\ = \frac{2m}{\pi a b} \frac{1}{8} \pi a^3 b = \frac{1}{4} m a^2$$

$$\bar{I}_{CC'}^{mass} = I_{AA'}^{mass} + I_{BB'}^{mass} \\ = \frac{1}{4} m a^2 + \frac{1}{4} m b^2 \left(1 - \frac{64}{9\pi^2} \right) = \frac{1}{4} m (a^2 + 0.279 b^2)$$

$$9.116 \quad m = \rho t A = \rho t [(2a)^2 - a^2] = \rho t (3a^2)$$

$$I^{mass} = \rho t I^{area} = \frac{m}{3a^2} I^{area}$$

$$(a) \quad I_{BB'}^{area} = \frac{1}{3} (2a)(2a)^3 - \frac{1}{3} (a)(a)^3 = 5a^4$$

$$I_{BB'}^{mass} = \frac{m}{3a^2} I_{BB'}^{area} \\ = \frac{m}{3a^2} 5a^4 = \frac{5}{3} m a^2$$

$$(b) \quad \bar{X} = \frac{\Sigma(\bar{y}A)}{\Sigma A} = \frac{a[(2a)^2] + \frac{a}{2}(-a^2)}{(2a)^2 - a^2} = \frac{7}{6} a$$

$$\bar{Y} = \bar{X} = \frac{7}{6} a, \quad I_{AA'}^{mass} = I_{BB'}^{mass} = \frac{5}{3} m a^2$$

$$I_O^{mass} = I_{AA'}^{mass} + I_{BB'}^{mass} \\ = 2 \left(\frac{5}{3} m a^2 \right) = \frac{10}{3} m a^2$$

$$\bar{I}_{CC'}^{mass} = I_O^{mass} - m d^2 = I_O^{mass} - m (\bar{X}^2 + \bar{Y}^2) \\ = \frac{10}{3} m a^2 - m \left[\left(\frac{7}{6} a \right)^2 + \left(\frac{7}{6} a \right)^2 \right] = \frac{11}{18} m a^2$$

$$9.117 \quad m = \rho t A = \rho t (2ab)$$

$$I^{mass} = \rho t I^{area} = \frac{m}{2ab} I^{area}$$

$$(a) \quad \bar{I}_x^{area} = 2 \left[\frac{1}{12} (2b)(a)^3 \right] = \frac{1}{3} a^3 b$$

$$\begin{aligned} \bar{I}_x^{mass} &= \frac{m}{2ab} \bar{I}_x^{area} \\ &= \frac{m}{2ab} \frac{1}{3} a^3 b = \frac{1}{6} m a^2 \end{aligned}$$

$$(b) \quad \bar{I}_{z'}^{mass} = \frac{1}{6} m b^2$$

$$I_{CC'}^{mass} = \bar{I}_x^{mass} + \bar{I}_{z'}^{mass} = \frac{1}{6} m a^2 + \frac{1}{6} m b^2$$

$$\begin{aligned} I_y^{mass} &= I_{CC'}^{mass} + m b^2 \\ &= \frac{1}{6} m a^2 + \frac{1}{6} m b^2 + m b^2 = \frac{1}{6} m a^2 + \frac{7}{6} m b^2 = \frac{1}{6} m (a^2 + 7b^2) \end{aligned}$$