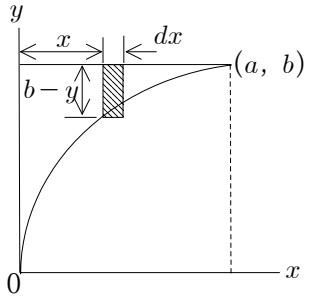
정 역 학　　연습문제 풀이　　제9장

<9.1~9.5절>

9.2　$y = k(x-a)^2 + c$

$(a,\ b) \quad \Rightarrow \quad b = c$

$(0,\ 0) \quad \Rightarrow \quad 0 = ka^2 + c \quad \Rightarrow \quad k = -\dfrac{b}{a^2}$



$\Rightarrow \quad y = -\dfrac{b}{a^2}(x-a)^2 + b$

$b - y = \dfrac{b}{a^2}(x-a)^2 = \dfrac{b}{a^2}(x^2 - 2ax + a^2)$

$dI_y = x^2 dA = x^2(b-y)dx$

$= \dfrac{b}{a^2}(x^4 - 2ax^3 + a^2x^2)dx$

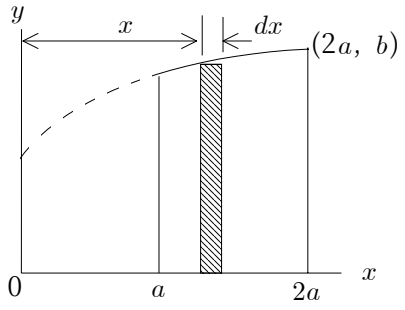
$I_y = \int dI_y = \int_0^a \dfrac{b}{a^2}(x^4 - 2ax^3 + a^2x^2)dx$

$= \dfrac{b}{a^2}\left[\dfrac{1}{5}x^5 - \dfrac{a}{2}x^4 + \dfrac{a^2}{3}x^3\right]_0^a = ba^3\left(\dfrac{1}{5} - \dfrac{1}{2} + \dfrac{1}{3}\right) = \dfrac{a^3 b}{30}$　　(교재 답 오류)

9.10　$x = ky^3$

$(2a,\ b) \quad \Rightarrow \quad 2a = kb^3 \quad \Rightarrow \quad k = \dfrac{2a}{b^3}$

$\Rightarrow \quad x = \dfrac{2a}{b^3}y^3 \quad$ 또는 $\quad y^3 = \dfrac{b^3}{2a}x$



$dI_x = \dfrac{1}{3}y^3 dx = \dfrac{1}{3}\dfrac{b^3}{2a}x\,dx$

$I_x = \int dI_x = \int_a^{2a} \dfrac{1}{3}\dfrac{b^3}{2a}x\,dx$

$= \dfrac{b^3}{6a}\left[\dfrac{1}{2}x^2\right]_a^{2a} = \dfrac{b^3}{12a}[(2a)^2 - a^2] = \dfrac{1}{4}ab^3$

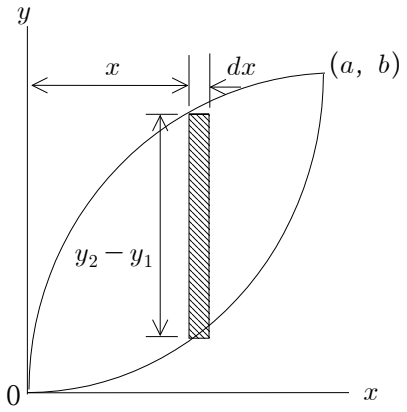
9.18

$$y_1 = k_1 x^4 \qquad y_2 = k_2 x^{1/4}$$

$$(a,\ b) \quad \Rightarrow \qquad b = k_1 a^4 \qquad b = k_2 a^{1/4}$$

$$k_1 = \frac{b}{a^4} \qquad k_2 = \frac{b}{a^{1/4}}$$

$$\Rightarrow \qquad y_1 = \frac{b}{a^4}\, x^4 \qquad y_2 = \frac{b}{a^{1/4}}\, x^{1/4}$$



$$dA = (y_2 - y_1)\,dx = \left(\frac{b}{a^{1/4}}\, x^{1/4} - \frac{b}{a^4}\, x^4\right) dx$$

$$A = \int dA = \int_0^a \left(\frac{b}{a^{1/4}}\, x^{1/4} - \frac{b}{a^4}\, x^4\right) dx$$

$$= \left[\frac{b}{a^{1/4}}\,\frac{4}{5} x^{5/4} - \frac{b}{a^4}\,\frac{1}{5} x^5\right]_0^a = \frac{3}{5}\, a\, b$$

$$dI_y = x^2\, dA = x^2 \left(\frac{b}{a^{1/4}}\, x^{1/4} - \frac{b}{a^4}\, x^4\right) dx = \left(\frac{b}{a^{1/4}}\, x^{9/4} - \frac{b}{a^4}\, x^6\right) dx$$

$$I_y = \int dI_y = \int_0^a \left(\frac{b}{a^{1/4}}\, x^{9/4} - \frac{b}{a^4}\, x^6\right) dx$$

$$= \left[\frac{b}{a^{1/4}}\,\frac{4}{13}\, x^{13/4} - \frac{b}{a^4}\,\frac{1}{7}\, x^7\right]_0^a = \left(\frac{4}{13} - \frac{1}{7}\right) a^3 b = \frac{15}{91}\, a^3 b$$

$$k_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{\frac{15}{91}\, a^3 b}{\frac{3}{5}\, a\, b}} = \sqrt{\frac{25}{91}\, a^2} = \frac{5}{\sqrt{91}}\, a = 0.524\ a$$