공 학 수 학 보충문제 (제9,10장) 정 답

출처 : Kreyszig 공업수학, 개정8판, 문제풀이 해설, 범한서적㈜, 2001.

9-1 함수의 직교성

■ 복습문제(4장)

26. $\frac{1}{\sqrt{\pi}}$, $\sqrt{\frac{2}{\pi}}\cos x$, $\sqrt{\frac{2}{\pi}}\cos 2x$, $\sqrt{\frac{2}{\pi}}\cos 3x$, $\cdots$

27. $\sqrt{\frac{\omega}{\pi}}\sin\omega x$, $\sqrt{\frac{\omega}{\pi}}\sin 2\omega x$, $\sqrt{\frac{\omega}{\pi}}\sin 3\omega x$, $\cdots$

28. $\frac{1}{2}$, $\sqrt{\frac{3}{2}}x$, $\sqrt{\frac{5}{8}}(3x^2-1)$, $\sqrt{\frac{7}{8}}(5x^3-3x)$

29. $\sqrt{\frac{2}{\pi}}$, $\frac{2}{\sqrt{\pi}}\cos 4nx$, $\frac{2}{\sqrt{\pi}}\sin 4nx$ $\quad (n = 1, 2, \cdots)$

9-3 Sturm-Liouville 문제와 고유함수

■ 연습문제 4.7

3. $\lambda = \left[(n+\frac{1}{2})\pi\right]^2$ $\quad (n = 0, 1, 2, 3, \cdots)$, $\qquad y_n(x) = \sin(n+\frac{1}{2})\pi x$

4. $\lambda = \left[\frac{(n+1)\pi}{L}\right]^2$ $\quad (n = 0, 1, 2, 3, \cdots)$, $\qquad y_n(x) = \sin\frac{(n+1)\pi x}{L}$

5. $\lambda = \left[\frac{(n+\frac{1}{2})\pi}{L}\right]^2$ $\quad (n = 0, 1, 2, 3, \cdots)$, $\qquad y_n(x) = \sin\frac{(n+\frac{1}{2})\pi x}{L}$

6. $\lambda = n^2$ $\quad (n = 0, 1, 2, 3, \cdots)$,
   $y_0(x) = 1$,
   $n = 1, 2, 3, \cdots$에 대해서는 $y_{n1}(x) = \cos nx$, $\quad y_{n2}(x) = \sin nx$

■ 복습문제(4장)

30. $\lambda = \nu^2$ $(\nu = 1, 2, 3, \cdots)$, $\qquad y_\nu(x) = \sin\nu x$

31. $\lambda = \left(\frac{n\pi}{L}\right)^2$ $\quad (n = 0, 1, 2, 3, \cdots)$,
   $y_0(x) = 1$,
   $n = 1, 2, 3, \cdots$에 대해서는 $y_{n1}(x) = \cos\frac{n}{L}x$, $\quad y_{n2}(x) = \sin\frac{n}{L}x$

32. $\lambda = \alpha_{n1}$ $\quad$ ($J_1(x)$ 의 n번째 양수 zero), $\quad y_n(x) = J_1(\alpha_{n1}x)$ $\quad (n = 1, 2, 3, \cdots)$

33. $\lambda = \nu^2$ $(\nu = 1, 2, 3, \cdots)$,
   ν가 홀수일 때 $y_\nu(x) = \cos\nu x$, $\quad \nu$가 짝수일 때 $y_\nu(x) = \sin\nu x$

9-5 Fourier 급수

■ 연습문제 10.1

1. 2π, 2π, π, π, 2, 2, 1, 1

2. $\frac{2\pi}{n}$, $\frac{2\pi}{n}$, k, k, $\frac{k}{n}$, $\frac{k}{n}$

7~18. (그래프)

■ 연습문제 10.2

1. $a_0 = 1,\quad a_n\ ;\ a_{4m+1} = \dfrac{2}{n\pi},\quad a_{4m+2} = 0,\quad a_{4m+3} = \dfrac{-2}{n\pi},\quad a_{4m+4} = 0,\quad b_n = 0$

$f(x) = \dfrac{1}{2} + \dfrac{2}{\pi}(\cos x - \dfrac{1}{3}\cos 3x + \dfrac{1}{5}\cos 5x + \cdots)$

2. $a_0 = \dfrac{1}{2},\quad a_n\ ;\ a_{4m+1} = \dfrac{1}{n\pi},\quad a_{4m+2} = 0,\quad a_{4m+3} = \dfrac{-1}{n\pi},\quad a_{4m+4} = 0$

$b_n\ ;\ b_{4m+1} = \dfrac{1}{n\pi},\quad b_{4m+2} = \dfrac{2}{n\pi},\quad b_{4m+3} = \dfrac{1}{n\pi},\quad a_{4m+4} = 0$

$f(x) = \dfrac{1}{4} + \dfrac{1}{\pi}(\cos x - \dfrac{1}{3}\cos 3x + \dfrac{1}{5}\cos 5x + \cdots)$

$+ \dfrac{1}{\pi}(\sin x + \sin 2x + \dfrac{1}{3}\sin 3x + \dfrac{1}{5}\sin 5x + \dfrac{1}{3}\sin 6x + \cdots)$

3. $a_0 = k,\quad a_n = 0,\quad b_n\ ;\ b_{2m+1} = \dfrac{2k}{n\pi},\quad b_{2m+2} = 0$

$f(x) = \dfrac{k}{2} + \dfrac{2k}{\pi}(\sin x + \dfrac{1}{3}\sin 3x + \dfrac{1}{5}\sin 5x + \cdots)$

4. $a_0 = 0,\quad a_n\ ;\ a_{4m+1} = \dfrac{-2}{n\pi},\quad a_{4m+2} = 0,\quad a_{4m+3} = \dfrac{2}{n\pi},\quad a_{4m+4} = 0$

$b_n\ ;\ b_{4m+1} = 0,\quad b_{4m+2} = \dfrac{4}{n\pi},\quad b_{4m+3} = 0,\quad a_{4m+4} = 0$

$f(x) = -\dfrac{2}{\pi}(\cos x - \dfrac{1}{3}\cos 3x + \dfrac{1}{5}\cos 5x + \cdots)$

$+ \dfrac{2}{\pi}(\sin 2x + \dfrac{1}{3}\sin 3x + \dfrac{1}{5}\sin 5x + \cdots)$

5. $a_0 = 0,\quad a_n = 0,\quad b_n\ ;\ b_{2m+1} = \dfrac{2}{n},\quad b_{2m+2} = \dfrac{-2}{n}$

$f(x) = 2\ (\sin x - \dfrac{1}{2}\sin 2x + \dfrac{1}{3}\sin 3x + \cdots)$

6. $a_0 = 2\pi,\quad a_n = 0,\quad b_n = \dfrac{-2}{n}$

$f(x) = \pi - 2\ (\sin x + \dfrac{1}{2}\sin 2x + \dfrac{1}{3}\sin 3x + \cdots)$

7. $a_0 = \dfrac{2\pi^2}{3},\quad a_n\ ;\ a_{2m+1} = \dfrac{-4}{n^2},\quad a_{2m+2} = \dfrac{4}{n^2},\quad b_n = 0$

$f(x) = \dfrac{\pi^2}{3} - 4\ (\cos x - \dfrac{1}{4}\cos 2x + \dfrac{1}{9}\cos 3x + \cdots)$

8. $a_0 = \dfrac{8\pi^2}{3},\quad a_n = \dfrac{4}{n^2},\quad b_n = (-1)^n\dfrac{4\pi}{n}$

$f(x) = \dfrac{4\pi^2}{3} + 4\ (\cos x + \dfrac{1}{4}\cos 2x + \dfrac{1}{9}\cos 3x + \cdots)$

$- 4\pi\ (\sin x - \dfrac{1}{2}\sin 2x + \dfrac{1}{3}\sin 3x + \cdots)$

9. $a_0 = 0,\quad a_n = 0,\quad b_n = 2\left[\dfrac{\pi^2(-1)^{n+1}}{n} + \dfrac{6(-1)^n}{n^3}\right]$

$f(x) = 2\ [(\pi^2 - 6)\sin x - (\dfrac{\pi^2}{2} - \dfrac{6}{2^3})\sin 2x + (\dfrac{\pi^2}{3} - \dfrac{6}{3^3})\sin 3x + \cdots]$

10. $a_0 = \pi$, a_n ; $a_{2m+1} = \frac{-4}{n^2\pi}$, $a_{2m+2} = 0$, $b_n = \frac{2(-1)^{n+1}}{n}$

$$f(x) = \frac{\pi}{2} - \frac{4}{\pi}\left(\cos x + \frac{1}{9}\cos 3x + \frac{1}{25}\cos 5x + \cdots\right)$$
$$+ 2\left(\sin x - \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x + \cdots\right)$$

11. $a_0 = 0$, $a_n = 0$, b_n ; $b_{2m+1} = \frac{-4}{n\pi}$, $b_{2m+2} = 0$

$$f(x) = -\frac{4}{\pi}\left(\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \cdots\right)$$

12. $a_0 = \frac{-1}{2}$, a_n ; $a_{4m+1} = \frac{-1}{n\pi}$, $a_{4m+2} = 0$, $a_{4m+3} = \frac{1}{n\pi}$, $a_{4m+4} = 0$

b_n ; $b_{4m+1} = \frac{-1}{n\pi}$, $b_{4m+2} = \frac{-2}{n\pi}$, $b_{4m+3} = \frac{-1}{n\pi}$, $a_{4m+4} = 0$

$$f(x) = -\frac{1}{4} - \frac{1}{\pi}\left(\cos x - \frac{1}{3}\cos 3x + \frac{1}{5}\cos 5x + \cdots\right)$$
$$- \frac{1}{\pi}\left(\sin x + \sin 2x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \cdots\right)$$

13. $a_0 = 0$, a_n ; $a_{4m+1} = \frac{4}{n\pi}$, $a_{4m+2} = 0$, $a_{4m+3} = \frac{-4}{n\pi}$, $a_{4m+4} = 0$, $b_n = 0$

$$f(x) = \frac{4}{\pi}\left(\cos x - \frac{1}{3}\cos 3x + \frac{1}{5}\cos 5x + \cdots\right)$$

14. $a_0 = 0$, $a_n = 0$, b_n ; $b_{4m+1} = \frac{4}{n^2\pi}$, $b_{4m+2} = 0$, $b_{4m+3} = \frac{-4}{n^2\pi}$, $a_{4m+4} = 0$

$$f(x) = \frac{4}{\pi}\left(\sin x - \frac{1}{9}\sin 3x + \frac{1}{25}\sin 5x + \cdots\right)$$

15. $a_0 = 0$, $a_n = 0$, b_n ; $b_{4m+1} = \frac{2}{n^2\pi}$, $b_{4m+2} = \frac{1}{n}$, $b_{4m+3} = \frac{-2}{n^2\pi}$, $a_{4m+4} = \frac{-1}{n}$

$$f(x) = \frac{2}{\pi}\sin x + \frac{1}{2}\sin 2x - \frac{2}{9\pi}\sin 3x - \frac{1}{4}\sin 4x + \cdots)$$

16. $a_0 = \frac{\pi^2}{3}$, a_n ; $a_{4m+1} = \frac{-4}{n^3\pi}$, $a_{4m+2} = \frac{-2}{n^2}$, $a_{4m+3} = \frac{4}{n^3\pi}$, $a_{4m+4} = \frac{2}{n^2}$, $b_n = 0$

$$f(x) = \frac{\pi^2}{6} - \frac{4}{\pi}\cos x - \frac{1}{2}\cos 2x + \frac{4}{27\pi}\cos 3x + \frac{1}{8}\cos 4x - \frac{4}{125\pi}\cos 5x + \cdots$$

10-1 Fourier 급수

■ 연습문제 10.3

1. $a_0 = 0$, $a_n = 0$, $b_n = \frac{2[1-(-1)^n]}{n\pi}$

$$f(x) = \frac{4}{\pi}\left(\sin \pi x + \frac{1}{3}\sin 3\pi x + \frac{1}{5}\sin 5\pi x + \cdots\right)$$

2. $a_0 = 0$, $a_n = 0$, $b_n = -\frac{2[1-(-1)^n]}{n\pi}$

$$f(x) = -\frac{4}{\pi}\left(\sin \pi x + \frac{1}{3}\sin 3\pi x + \frac{1}{5}\sin 5\pi x + \cdots\right)$$

3. $a_0 = 2$, $a_n = 0$, $b_n = \frac{2[1-(-1)^n]}{n\pi}$

$$f(x) = 1 + \frac{4}{\pi}\left(\sin\frac{\pi}{2}x + \frac{1}{3}\sin\frac{3}{2}\pi x + \frac{1}{5}\sin\frac{5}{2}\pi x + \cdots\right)$$

4. $a_0 = 2,\quad a_n = -\frac{4[1-(-1)^n]}{n^2\pi^2},\quad b_n = 0$

$$f(x) = 1 - \frac{8}{\pi^2}\left(\cos\frac{\pi}{2}x + \frac{1}{9}\cos\frac{3}{2}\pi x + \frac{1}{25}\cos\frac{5}{2}\pi x + \cdots\right)$$

5. $a_0 = 0,\quad a_n = 0,\quad b_n = \frac{(-1)^{n+1}}{n\pi}$

$$f(x) = \frac{4}{\pi}\left(\sin\pi x - \frac{1}{2}\sin 2\pi x + \frac{1}{3}\sin 3\pi x + \cdots\right)$$

6. $a_0 = \frac{4}{3},\quad a_n = -\frac{4(-1)^n}{n^2\pi^2},\quad b_n = 0$

$$f(x) = \frac{2}{3} + \frac{4}{\pi^2}\left(\cos\pi x - \frac{1}{4}\cos 2\pi x + \frac{1}{9}\cos 3\pi x + \cdots\right)$$

7. $a_0 = 2,\quad a_n = \frac{12(-1)^n}{n^2\pi^2},\quad b_n = 0$

$$f(x) = 1 - \frac{12}{\pi^2}\left(\cos\pi x - \frac{1}{4}\cos 2\pi x + \frac{1}{9}\cos 3\pi x + \cdots\right)$$

8. $a_0 = \frac{1}{2},\quad a_n = \frac{1-(-1)^n}{n^2\pi^2},\quad b_n = 0$

$$f(x) = \frac{1}{4} + \frac{2}{\pi^2}\left(\cos 2\pi x + \frac{1}{9}\cos 6\pi x + \frac{1}{25}\cos 10\pi x + \cdots\right)$$

9. $a_0 = \frac{1}{2},\quad a_n = \frac{-1+(-1)^n}{n^2\pi^2},\quad b_n = \frac{(-1)^n}{n\pi}$

$$f(x) = \frac{1}{4} - \frac{2}{\pi^2}\left(\cos\pi x + \frac{1}{9}\cos 3\pi x + \frac{1}{25}\cos 5\pi x + \cdots\right)$$
$$+ \frac{1}{\pi}\left(\sin\pi x - \frac{1}{2}\sin 2\pi x + \frac{1}{3}\sin 3\pi x + \cdots\right)$$

10. $a_0 = 0,\quad a_n = -\frac{2[1-(-1)^n]}{n^2\pi^2},\quad b_n = \frac{1-(-1)^n}{n\pi}$

$$f(x) = -\frac{4}{\pi^2}\left(\cos\pi x + \frac{1}{9}\cos 3\pi x + \frac{1}{25}\cos 5\pi x + \cdots\right)$$
$$+ \frac{2}{\pi}\left(\sin\pi x + \frac{1}{3}\sin 3\pi x + \frac{1}{5}\sin 5\pi x + \cdots\right)$$

11. $a_0 = 4,\quad a_n = \frac{4n}{4n^2-1},\quad b_n = 0$

$$f(x) = 2 - 4\left(\frac{1}{1\cdot 3}\cos 2\pi x + \frac{1}{3\cdot 5}\cos 4\pi x + \frac{1}{5\cdot 7}\cos 6\pi x + \cdots\right)$$

12. $a_0 = 0,\quad a_n = 0,\quad b_n = (-1)^{n+1}\left(\frac{1}{n} - \frac{6}{n^3\pi^2}\right)$

$$f(x) = \left(1-\frac{6}{\pi^2}\right)\sin\pi x - \left(\frac{1}{2}-\frac{6}{2^3\pi^2}\right)\sin 2\pi x + \left(\frac{1}{3}-\frac{6}{3^3\pi^2}\right)\sin 3\pi x + \cdots$$