공 학 수 학 보충문제 (제3장) 정 답

출처 : Kreyszig 공업수학, 개정8판, 문제풀이 해설, 범한서적㈜, 2001.

3-1 멱급수와 해석함수

■ 연습문제 4.2

13. $R = \infty$
14. $R = 1$
15. $R = \sqrt{3}$
16. $R = 1$
17. $R = \sqrt{k}$
18. $R = \infty$
19. $R = \sqrt{3/2}$
20. $R = 0$
21. $R = \sqrt{2}$
23. $\sum_{m=1}^{\infty} \frac{(m+1)(m+2)}{(m+1)^2+1} x^m, \quad R = 1$
24. $\sum_{m=0}^{\infty} (m+2)(m+1) x^m, \quad R = 1$
25. $\sum_{m=1}^{\infty} \frac{m^3}{(m+1)!} x^m, \quad R = \infty$

3-2 멱급수에 의한 미분방정식의 해

■ 연습문제 4.1

1. $y = A_0 (1 + 3x + \frac{3^2}{2}x^2 + \frac{3^3}{3!}x^3 + \frac{3^4}{4!}x^4 + \cdots)$
2. $y = A_0 (1 + kx + \frac{k^2}{2}x^2 + \frac{k^3}{3!}x^3 + \frac{k^4}{4!}x^4 + \cdots)$
3. $y = A_0 [1 - 2x + \frac{(-2)^2}{2}x^2 + \frac{(-2)^3}{3!}x^3 + \frac{(-2)^4}{4!}x^4 + \cdots]$
4. $y = A_0 (1 + x + x^2 + x^3 + x^4 + \cdots)$
5. $y = A_0 (1 + x^2 + \frac{1}{2}x^4 + \frac{1}{3!}x^6 + \cdots)$
6. $y = A_0 (1 + x)$
7. $y = A_0 (1 + \frac{1}{2}x^2 + \frac{1}{4}\frac{1}{2}x^4 + \frac{1}{8}\frac{1}{3!}x^6 + \cdots)$
8. $y = A_0 [1 + \frac{1}{2}(2x)^2 + \frac{1}{4!}(2x)^4 + \frac{1}{6!}(2x)^6 + \cdots] + A_1 [2x + \frac{1}{3!}(2x)^3 + \frac{1}{5!}(2x)^5 + \cdots]$
9. $y = A_0 [1 - \frac{1}{2}(3x)^2 + \frac{1}{4!}(3x)^4 - \frac{1}{6!}(3x)^6 + \cdots] + A_1 [3x - \frac{1}{3!}(3x)^3 + \frac{1}{5!}(3x)^5 - \cdots]$
10. $y = A_0 (1 + x + \frac{1}{2}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \cdots) + A_1$
11. $y = A_0 (1 + x^3 + \frac{1}{2}x^6 + \frac{1}{3!}x^9 + \cdots)$
12. $y = A_0 (1 + \frac{1}{2}x^2 + \frac{1}{4!}x^4 + \frac{1}{6!}x^6 + \cdots) + A_1 (x + \frac{1}{3!}x^3 + \frac{1}{5!}x^5 + \cdots)$

■ 연습문제 4.2

1. $y = A_0 \sum_{k=0}^{\infty} \frac{(-x^2)^k}{k!}$
2. $y = A_0 (1 - \frac{1}{4}x^2 - \frac{1}{12}x^3 + \frac{1}{120}x^5 + \frac{1}{288}x^6 + \cdots)$

3. $y = -\frac{k}{3} + A_3 x^3$

4. $y = A_0 (1 + x^2 + x^4 + x^6 + \cdots)$

5. $y = A_0 + A_1 x + (\frac{3}{2} A_1 - A_0) x^2 + (\frac{7}{6} A_1 - A_0) x^3 + \cdots$

6. $y = (A_0 + A_1 x)(1 + x^2 + \frac{1}{2} x^4 + \frac{1}{3!} x^6 + \cdots)$

7. $y = A_0 \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} (2x)^{2k} + A_1 \sum_{k=0}^{\infty} \frac{(-4)^k}{(2k+1)!} x^{2k+1}$

8. $y = A_0 (1 - x^2 - \frac{1}{3} x^4 - \frac{1}{5} x^6 - \cdots) + A_1 x$

9. $y = A_0 \sum_{n=0}^{\infty} \frac{k^n}{n!} (x-1)^n$

10. $y = A_0 [1 + \frac{1}{2}(x-1)^2 + \frac{1}{4!}(x-1)^4 + \cdots] + A_1 [(x-1) + \frac{1}{3!}(x-1)^3 + \frac{1}{5!}(x-1)^5 + \cdots]$

11. $y = A_0 x + x \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} (x-1)^n$

12. $\frac{1}{x}$은 $x = 0$에서 해석적이지 않음.

3-3 Legendre방정식

■ 연습문제 4.3

1. $y_1 = P_0(x), \quad y_2 = Q_0(x)$
2. $y_1 = P_1(x), \quad y_2 = Q_1(x)$
7. $y = c_1 P_3(\frac{x}{a}) + c_2 Q_3(\frac{x}{a})$

3-4 Bessel방정식

■ 연습문제 4.5

2. $y = c_1 J_5(x) + c_2 Y_5(x)$ (4.6절 문제임)
3. $y = c_1 J_{\frac{3}{2}}(5x) + c_2 J_{-\frac{3}{2}}(5x)$ (" ")

11. 13. 22. 23. 24. (확인)

25. $-J_0(x) - 2J_2(x) + c$

26. $x^3 J_1(x) - 2x^2 J_2(x) + c$

27. $-J_0(x) - 2J_2(x) - 2J_4(x) - c$

■ 연습문제 4.6

1. $y = c_1 J_{\frac{1}{2}}(x) + c_2 J_{-\frac{1}{2}}(x)$ (4.5절 문제임)
2. $y = c_1 J_\nu(\lambda x) + c_2 J_{-\nu}(\lambda x)$ ($\nu \neq$정수 일 때) (" ")