

기 계 진 동 학 도전문제 (제1장)

출처 및 해답 [수업지정도서 (중앙도서관 5층 디지털미디어룸 내)]

[1] S. G. Kelly, Fundamentals of Mechanical Vibrations, 2nd ed., McGraw-Hill, 2000.

[2] L. Meirovitch, Fundamentals of Vibrations, McGraw-Hill, 2001.

[3] 이시복 등 7인 공역, 기계진동학, 제6판, 퍼스트북, 2019.

(원서 : S. S. Rao, Mechanical Vibrations, Prentice Hall, 2017.)

1.0 진동학 개념

[1] Ex. 1.1

Example 1.1 Each of the systems of Fig. 1.3 is in equilibrium in the position shown and undergoes planar motion. All bodies are rigid. Specify, for each system, the number of degrees of freedom and recommend a set of generalized coordinates.

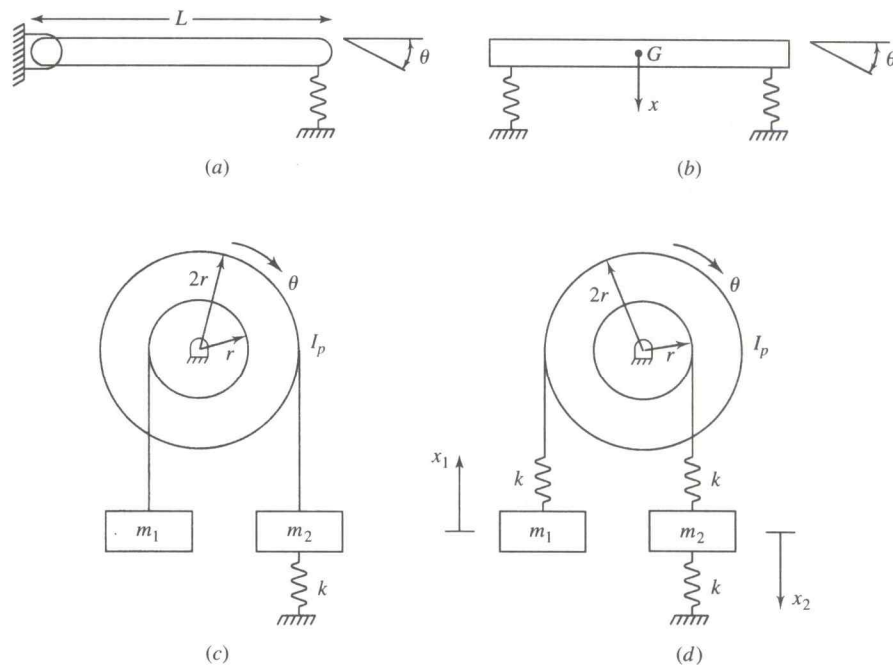


Figure 1.3 Systems for Example 1.1. One possible choice of a set of generalized coordinates is illustrated for each system.

Example 2.2 Derive the differential equation governing the angular oscillations of the compound pendulum of Fig. 2.3a.

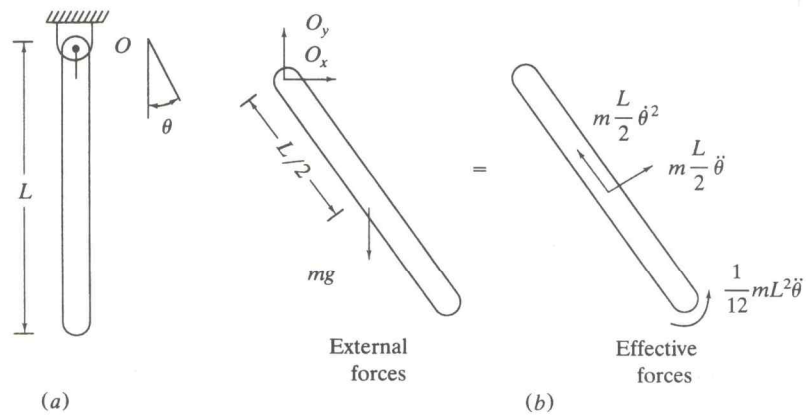


Figure 2.3 (a) Compound pendulum of Example 2.2 is a slender rod pinned at one end. The generalized coordinate θ is the counterclockwise angular displacement from equilibrium; (b) free-body diagrams at an arbitrary instant of time.

Example 1.18 Addition of Harmonic Motions

Find the sum of the two harmonic motion $x_1(t) = 10\cos\omega t$ and $x_2(t) = 15\cos(\omega t + 2)$.

Example 2.15 The schematic diagram of a large cannon is shown in Figure 2.31. When the gun is fired, high pressure gases accelerate the projectile inside the barrel to a very high velocity. the reaction force pushes the gun barrel in the opposite direction of the projectile. Since it is desirable to bring the gun barrel to rest in the shortest time without oscillation, it is made to translate backward against a critically damped spring-damper system called the recoil mechanism. In a particular case, the gun barrel and the recoil mechanism have a mass of 500 kg with a recoil spring of stiffness 10,000 N/m. The gun recoils 0.4 m upon firing. Find (1) the critical damping coefficient of the damper, (2) the initial recoil velocity of the gun, and (3) the time taken by the gun to return to a position 0.1 m from the initial position.

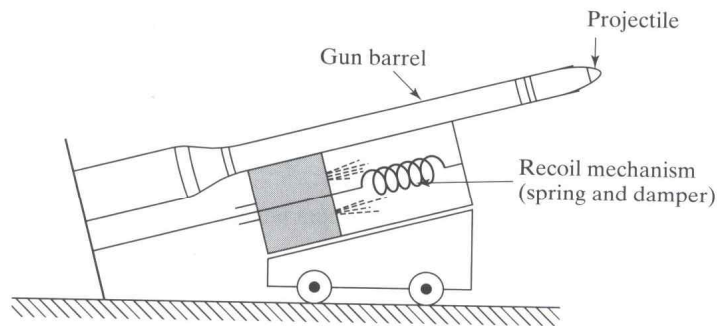


FIGURE 2. Recoil of cannon.

Example 2.19 The restroom door of Fig. 2.26 is equipped with a torsional spring and a torsional viscous damper so that it automatically returns to its closed position after being opened. The door has a mass of 60 kg and a centroidal moment of inertia about an axis parallel to the axis of the door's rotation of $7.2 \text{ kg}\cdot\text{m}^2$. The torsional spring has a stiffness of $25 \text{ N}\cdot\text{m}/\text{rad}$.

- What is the damping coefficient such that the system is critically damped?
- A man with an armload of packages, but in a hurry, kicks the door to cause it to open. What angular velocity must his kick impart to cause the door to open 70° ?
- How long after his kick will the door return to within 5° of completely closing?
- Repeat parts a-c if the door is designed with a damping ratio, $\zeta = 1.3$.

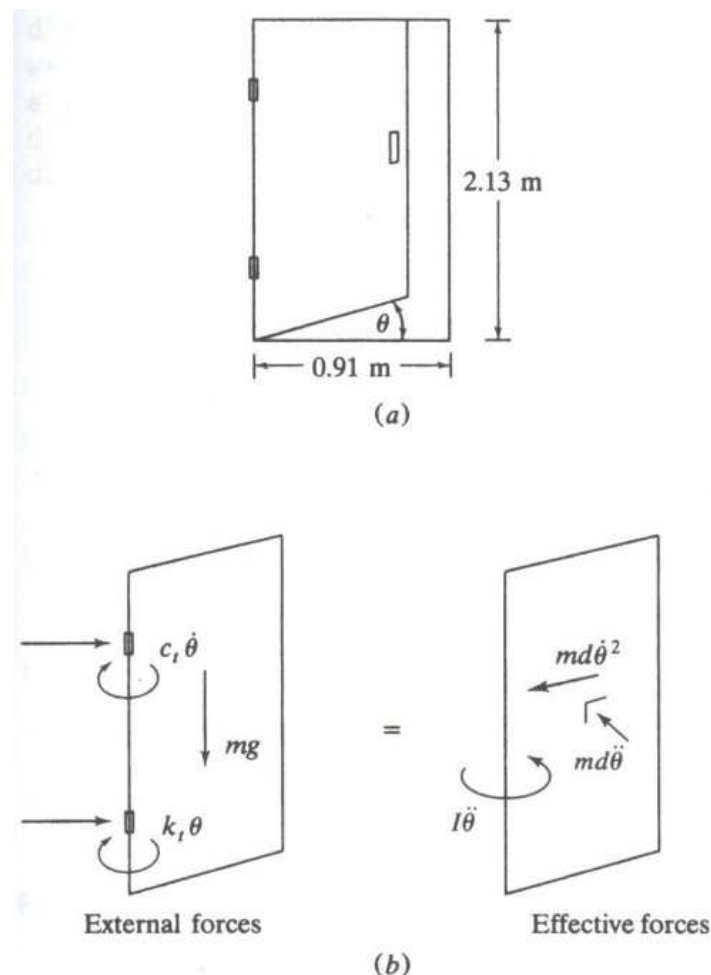


Figure 2.26

(a) The restroom door of Example 2.19 is modeled as a one-degree-of-freedom system with a torsional spring and a torsional viscous damper; (b) free-body diagrams of restroom door at an arbitrary instant.

Example 2.4 A slender rod of length L and mass m is pinned at O , as shown in Fig. 2.5. A spring of stiffness k is connected to the rod at point P while a dashpot of damping coefficient c is connected at point Q . Assuming small displacements, derive a linear differential equation governing the free vibrations of this system. Use x , the displacement of particle P , measured from the system's equilibrium position, as the generalized coordinate.

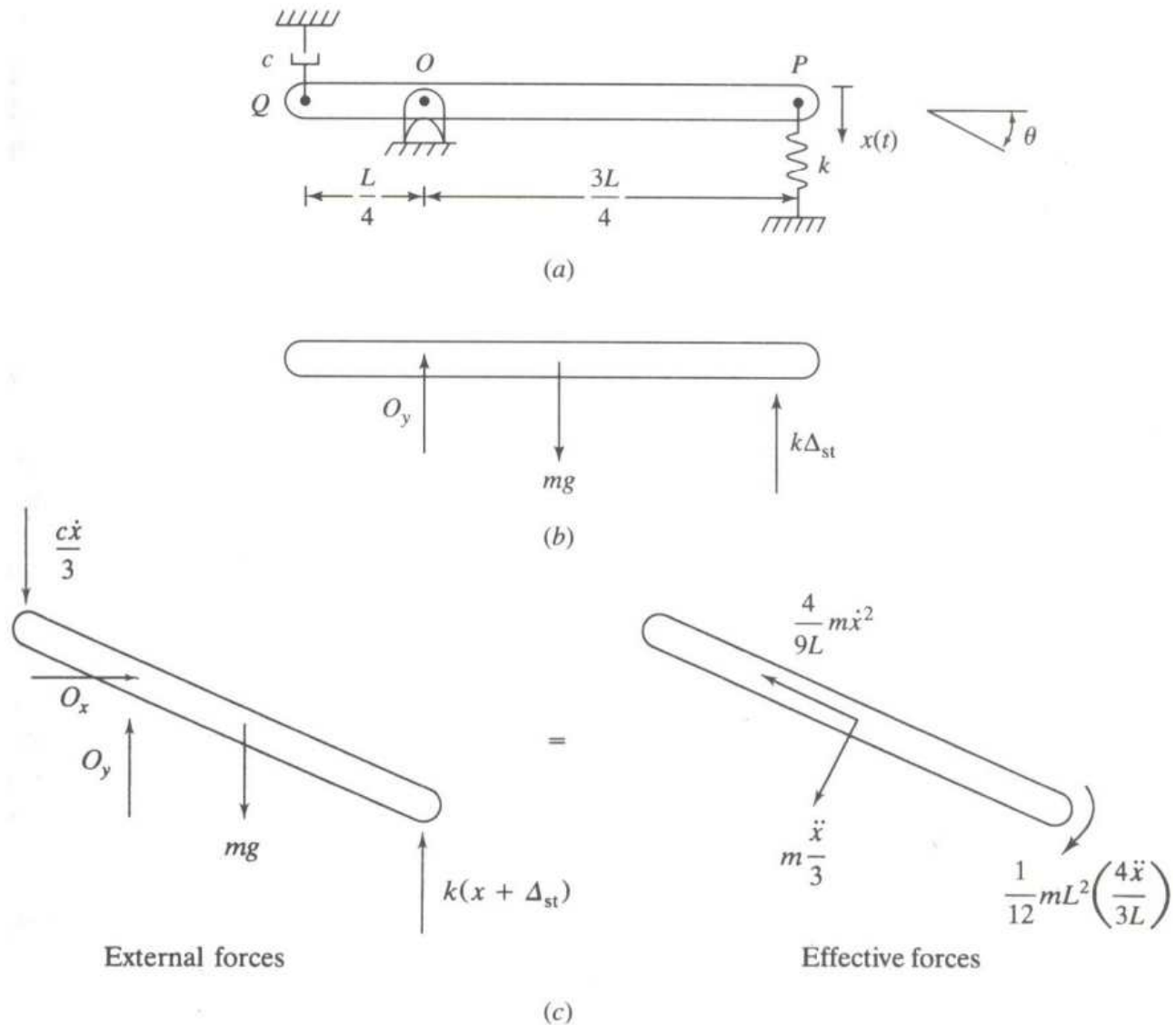


Figure 2.5 (a) System of Example 2.4; (b) free-body diagram of static equilibrium position; (c) free-body diagrams at an arbitrary instant.

Example 1.12 A cam-follower mechanism (Fig. 1.39) is used to convert the rotary motion of a shaft into the oscillating or reciprocating motion of a valve. The follower system consists of a pushrod of mass m_p , a rocker arm of mass m_r , and mass moment of inertia J_r about its C.G., a valve of mass m_v , and a valve spring of negligible mass. Find the equivalent mass (m_{eq}) of this cam-follower system by assuming the location of m_{eq} as (i) point A and (ii) point C .

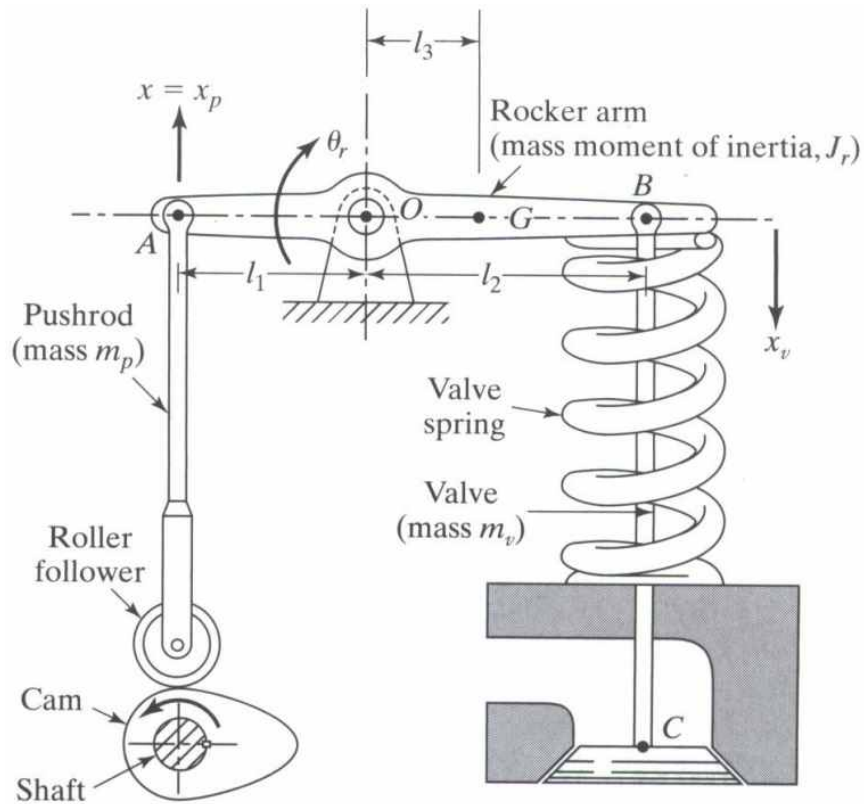


FIGURE 1. Cam-follower system.

Example 1.9 Model each of the system of Fig. 1.20 by a mass attached to a single spring of an equivalent stiffness. The system of Fig. 1.20c is to be modeled by a disk attached to a torsional spring of an equivalent stiffness.

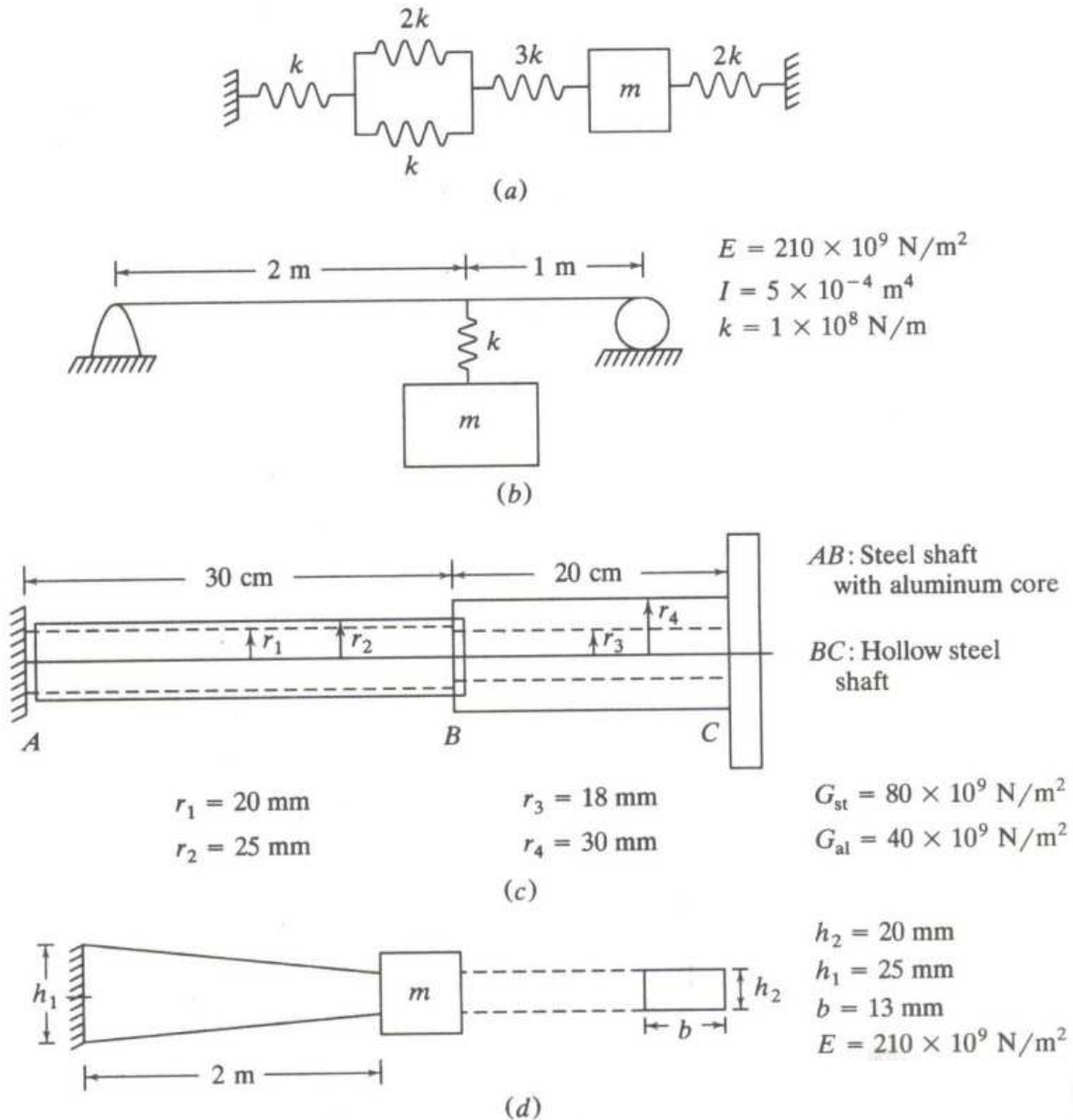


Figure 1.20 Systems for Example 1.9.

Example 1.8 Equivalent k of a Crane

The boom AB of the crane shown in Fig. 1.32(a) is a uniform steel bar of length 10 m and area of cross section $2,500 \text{ mm}^2$. A weight W is suspended while the crane is stationary. The cable $CDEBF$ is made of steel and has a cross-sectional area of 100 mm^2 . Neglecting the effect of the cable $CDEB$, find the equivalent spring constant of the system in the vertical direction.

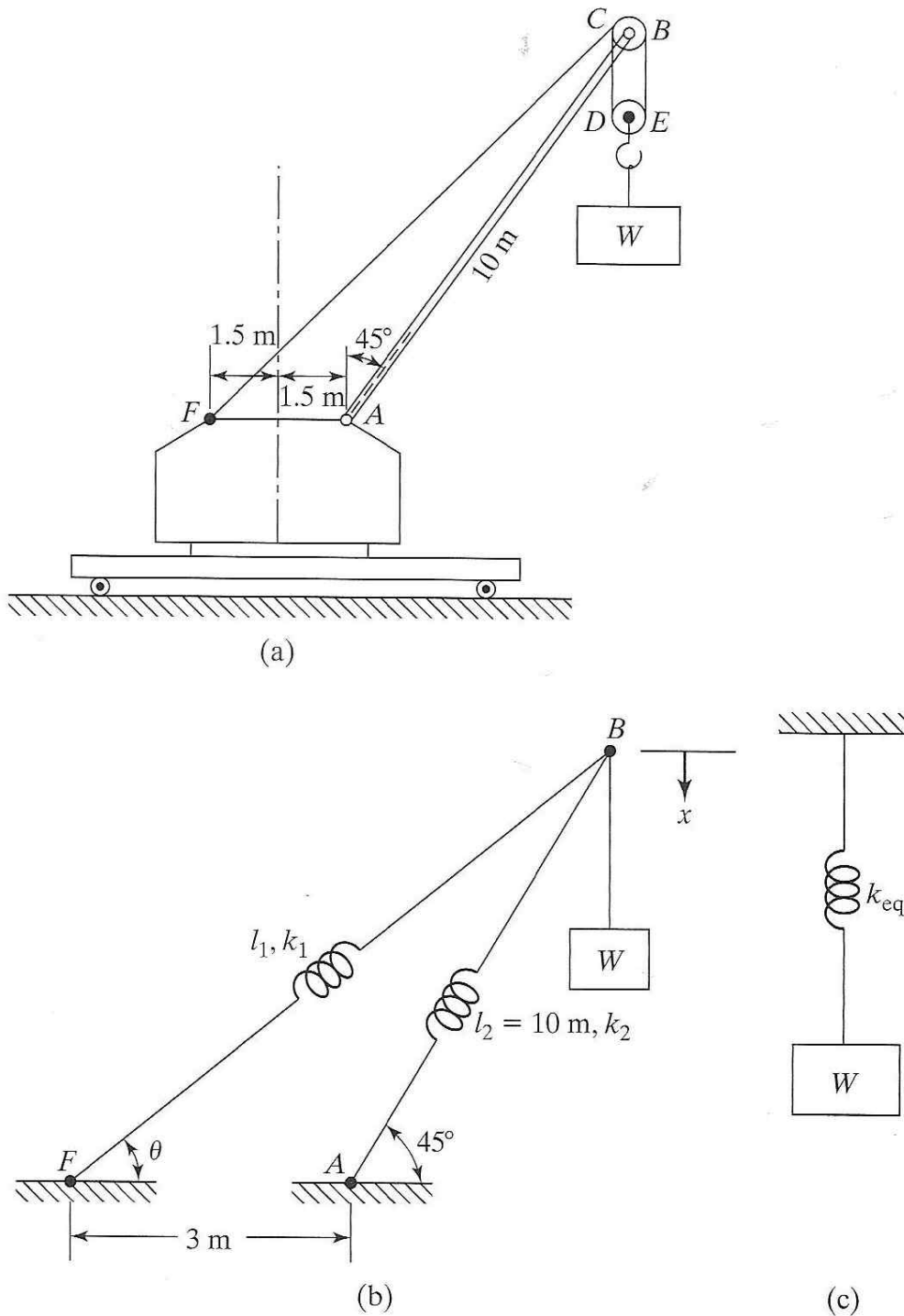


FIGURE 1. Crane lifting a load.

정답

1.0 진동학 개념

[1] Example 1.1

- (a) 1 DOF, θ
- (b) 2 DOF, x and θ
- (c) 1 DOF, θ
- (d) 3 DOF, x_1 , x_2 , and θ

1.1 자유진동

[1] Example 2.2

$$m \frac{L^2}{3} \ddot{\theta} + mg \frac{L}{2} \sin \theta = 0$$

1.2 조화운동

[3] Example 1.18

$$x(t) = 14.148 \cos(\omega t + 1.302)$$

1.3 점성감쇠

[3] Example 2.12

- (1) $c_{cr} = 4,470 \text{ N}\cdot\text{s/m}$
- (2) $\dot{x}_0 = 4.86 \text{ m/s}$
- (3) $t_2 = 0.826 \text{ s}$

[1] Example 2.19

- (a) $c_t = 44.1 \text{ N}\cdot\text{m}/(\text{rad/s})$
- (b) $\dot{\theta}_0 = 3.79 \text{ rad/s}$
- (c) $t = 4.66 \text{ s}$
- (d) $c_t = 57.4 \text{ N}\cdot\text{m}/(\text{rad/s})$, $\dot{\theta}_0 = 4.55 \text{ rad/s}$, $t = 6.20 \text{ s}$

1.4 모델링과 에너지방법

[1] Example 2.4

$$\frac{7m}{36} \ddot{x} + \frac{c}{12} \dot{x} + \frac{3k}{4} x = 0$$

[3] Example 1.12

$$m_{eq} = m_v + \frac{J_r}{l_2^2} + m_p \left(\frac{l_1}{l_2} \right)^2 + m_r \left(\frac{l_3}{l_2} \right)^2$$

1.5 강성

[1] Example 1.9

- (a) $k_{eq} = \frac{13}{5}k$
- (b) $k_{eq} = 70.2 \times 10^6 \text{ N/m}$
- (c) $k_{eq} = 0.101 \times 10^6 \text{ N}\cdot\text{m/rad}$
- (d) $k_{eq} = 30.6 \times 10^6 \text{ N/m}$

[3] Example 1.8

$$k_{eq} = 26.4 \times 10^6 \text{ N/m}$$